The Quantum-Classical Crossover in the Adiabatic Response of Chaotic Systems

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February 5, 2008

Abstract

The autocorrelation function of the force acting on a slow classical system, resulting from interaction with a fast quantum system is calculated following Berry-Robbins and Jarzynski within the leading order correction to the adiabatic approximation. The time integral of the autocorrelation function is proportional to the rate of dissipation. The fast quantum system is assumed to be chaotic in the classical limit for each configuration of the slow system. An analytic formula is obtained for the finite time integral of the correlation function, in the framework of random matrix theory (RMT), for a specific dependence on the adiabatically varying parameter. Extension to a wider class of RMT models is discussed. For the Gaussian unitary and symplectic ensembles for long times the time integral of the correlation function vanishes or falls off as a Gaussian with a characteristic time that is proportional to the Heisenberg time, depending on the details of the model. The fall off is inversely proportional to time for the Gaussian orthogonal ensemble. The correlation function is found to be dominated by the nearest neighbor level spacings. It was calculated for a variety of nearest neighbor level spacing distributions, including ones that do not originate from RMT ensembles. The various approximate formulas obtained are tested numerically in RMT. The results shed light on the quantum to classical crossover for chaotic systems. The implications on the possibility to experimentally observe deterministic friction are discussed.

PACS: 05.45.Mt, 05.45.Ac, 05.45.-a

1 Introduction

Dissipation of energy from a physical system to a thermal bath takes place as a result of a fluctuating force that acts on the system because of its coupling to the

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bath. The dissipative friction force is proportional to the correlation function of the fluctuating force. The force resulting from coupling to a chaotic system is also fluctuating. The question that will be studied in the present paper is on what time scales it leads to friction and what is the relation of this friction to the autocorrelation function of the fluctuating force. An example is slow particle coupled to a fast particle so that the motion of the fast particle is chaotic for each position of the slow particle.

Various models for dissipation of energy from a slow particle by a fast one have been developed. To our knowledge the first models of this type were introduced in the context of nuclear physics [1]. In particular, a model where particles move within a region bounded by a deforming boundary, modeling the nuclear surface was studied [2]. The energy transferred between the boundary and particles enclosed inside was calculated classically and quantum mechanically in the framework of some approximations. Recently some detailed numerical simulations were performed along these lines and the regime of validity of various approximations was tested [3]. The dissipation for a wide class of model systems was explored by Wilkinson and Austin [4-6] in the framework of random matrix theory (RMT), relevant for a situation where the motion of the fast particles is chaotic. The relation to Landau-Zener tunneling was also studied in these works. A different RMT model was studied by Mizutori and Aberg [7]. In all these studies dissipation was found. In addition, a different approach, aimed at emphasizing the relation to many body problems was introduced [8]. A unified picture of many of these models has recently been presented by Cohen [9]. A systematic investigation of the interaction of a slow system with a fast one is possible with the help of multiple scale analysis. Under such conditions Ott demonstrated [10] that the phase space volume enclosed by the energy surface of the fast particle is an adiabatic invariant, namely its change is much slower than that of the fast particle Hamiltonian. It has been demonstrated for various conditions that it is indeed an adiabatic invariant [11]. In the present paper we study the behavior of a slow particle that is coupled to a fast chaotic system. A model for such a system, that is quite general, and has been studied by Berry and Robbins (BR) [12] and by Jarzynski [13] in the framework of multiple scale analysis, is defined by the Hamiltonian:

$$\mathcal{H} = \frac{1}{2M}\mathbf{P}^2 + h\left(\mathbf{R}, \mathbf{z}\right). \tag{1}$$

The phase space coordinates of the slow particle are $(\mathbf{P}, \mathbf{R})$ and its mass is M. For simplicity it is coupled only through its position to the fast system whose phase space coordinates are $\mathbf{z} \equiv (\mathbf{p}, \mathbf{r})$. The latter system has the property that if $\mathbf{R}$ is kept fixed it is fully chaotic. The crucial feature of the system we wish to study in this work is that it exhibits a wide separation of time scales- the evolution of the fast system, characterized by the time scale T_{fast}, is so rapid that it explores all of the phase space accessible to it energetically before the slow particle, characterized by the time scale T_{slow}, moves appreciably.

The adiabaticity parameter is $\varepsilon \sim T_{\text{fast}}/T_{\text{slow}}$. One way to realize this is to couple two particles with a mass ratio of $m/M = \varepsilon^2 \ll 1$, as one can see by rescaling the equations of motion. We now turn to analyze the dynamics generated by the Hamiltonian (1) with the approximation that the slow particle evolves under the influence of the *average* force exerted on it by the fast system, which can be treated as a system described by a slowly varying Hamiltonian. The time dependence of this Hamiltonian is determined by the dynamics of the slow particle. First the classical dynamics is outlined and later the quantum mechanical behavior is summarized. In the case of (1) the average force is given by:

$$\mathbf{F}(\tau_a) = -\int \mathbf{dz}\ \rho\left(\mathbf{z}, \tau_a\right) \partial_{\mathbf{R}} h\left(\mathbf{z}, \mathbf{R}(\tau_a)\right), \tag{2}$$

where $\rho(\mathbf{z}, \tau_a)$ is a normalized probability density in the fast particle phase space. A formalism that includes *fluctuations* was developed by Jarzynski [13]. The results of this paper do not depend on these fluctuations and therefore the formalism of BR will be used. The probability density satisfies the Liouville equation:

$$\varepsilon \frac{\partial}{\partial \tau_a} \rho\left(\mathbf{z}, \tau_a\right) = \left\{ h\left(\mathbf{z}, \mathbf{R}(\tau_a)\right), \rho\left(\mathbf{z}, \tau_a\right) \right\}_{\mathbf{z}}, \tag{3}$$

written in a way that emphasizes that the evolution of the fast system is indeed on a much shorter time scale than the time scale on which the fast Hamiltonian changes. $\{\}_{\mathbf{z}}$ denotes Poisson brackets taken with respect to $\mathbf{z}$. With the aid of the multiple scale expansion:

$$\rho\left(\mathbf{z}, \tau_a\right) = \sum_{l=0}^{\infty} \varepsilon^l \rho_l\left(\mathbf{z}, \tau_a\right), \tag{4}$$

Berry and Robbins [12] were able to calculate the force acting on the slow particle up to first order in ε:

$$\mathbf{F} \approx \mathbf{F}_0 + \varepsilon \mathbf{F}_1. \tag{5}$$

To leading order, the force is given by the classical analogue of the Born-Oppenheimer force:

$$F_{0i}(\tau_a) = -\partial_{R_i} E\left(\mathbf{R}\right), \tag{6}$$

where $E\left(\mathbf{R}\right)$ is chosen such that the phase space volume enclosed by the energy surface of the fast particle, $\Omega\left(E\left(\mathbf{R}\right), \mathbf{R}\right)$, is constant. The leading correction to $\mathbf{F}_0$ includes a velocity dependent force:

$$F_{1i}(\tau_a) = -\sum_j K_{ij} \dot{R}_j; \qquad K_{ij} \equiv \Sigma^{-1} \partial_E \left[\Sigma(E, \mathbf{R}) \frac{1}{2} I_{ij}\left(E, \mathbf{R}\right) \right]_{E=E(\mathbf{R})}, \tag{7}$$

where $\Sigma(E,\mathbf{R}) \equiv \partial_E \Omega(E,\mathbf{R})$ and:

$$I_{ij}\left(E,\mathbf{R}\right) = 2\int_0^\infty dt'\; C_{ij}\left(E,\mathbf{R};t'\right); \tag{8}$$

$$C_{ij}\left(E,\mathbf{R};t'\right) \equiv \left\langle \partial_{R_i}\tilde{h}\left(\mathbf{z}_{t'}(\mathbf{z},\mathbf{R}),\mathbf{R}\right)\partial_{R_j}\tilde{h}\left(\mathbf{z},\mathbf{R}\right)\right\rangle_{E,\mathbf{R}}. \tag{9}$$

In the last equation:

$$\left\langle \ldots \right\rangle_{E,\mathbf{R}} \equiv \int \mathbf{dz}\;\rho_0\left(\mathbf{z},\tau_a\right)\ldots = \Sigma^{-1}(E,\mathbf{R})\int \mathbf{dz}\;\delta\left(E-h(\mathbf{z},\mathbf{R})\right)\ldots \tag{10}$$

denotes the microcanonical average and $\tilde{h}\left(\mathbf{z},\mathbf{R}\right) \equiv h\left(\mathbf{z},\mathbf{R}\right) - E(\mathbf{R})$. Finally, $\mathbf{z}_{t'}(\mathbf{z},\mathbf{R})$ in (9) is the classical trajectory obtained by integrating the equations of motion generated by $h\left(\mathbf{z},\mathbf{R}\right)$ with fixed $\mathbf{R}$ backwards from $\mathbf{z}$ for the time t'. In addition to the velocity dependent force (7), and to the same order in ε, there is a force that does not depend on velocity [14]. This force can be expressed as a gradient of a time dependent potential, and therefore is a correction to the Born-Oppenheimer force (6). Because of the form of $\mathbf{F}_1(\tau_a)$, it describes two qualitatively different forces. The first of these is geometric magnetism, and it is related to the antisymmetric part of K_{ij}. This force has been studied analytically in the systems under discussion by BR [12] and numerically by Berry and Sinclair [15]. The second force is associated with the symmetric part of K_{ij} and is related to deterministic friction. This force has been studied in [5, 16, 12, 13]. A central question that can be addressed at this point is under which conditions does the slow particle feel friction due to the velocity dependent force $\mathbf{F}_1$. In order for this to happen K_{ij} of (7) has to have a positive definite symmetric part.

The behavior when the fast system is quantum mechanical, to first order in ε, has also been studied by BR [12]. They found that in this case K_{ij} is an antisymmetric tensor, meaning that the system exhibits *only* geometric magnetism and *no* friction. This difference is a result of the discreteness of the quantum spectrum. The quantum correlation function corresponding to the symmetric part of C_{ij} of (9) is:

$$\begin{aligned}\frac{1}{2}\Big\{C_{ij}(n;t) + C_{ji}(n;t)\Big\} &= \\ = \sum_{m\neq n} \Re\left(\left\langle n\left|\partial_{R_i}\widehat{h}\right|m\right\rangle\left\langle m\left|\partial_{R_j}\widehat{h}\right|n\right\rangle\right)\cos\left[\frac{t}{\hbar}\left(E_n - E_m\right)\right],\end{aligned} \tag{11}$$

and the infinite time integral over it vanishes [12]. In order to understand how the crossover between the classical and quantum behavior occurs, it is instructive to calculate the integral of the correlation function over a finite time. Since the discordance between the quantum and the classical models appears in the

symmetric part of K_{ij} it can be studied for the case where $\mathbf{R}$ is replaced be a scalar, time dependent parameter, X. In this case $\frac{1}{2}\Big\{C_{ij}(n;t)+C_{ji}(n;t)\Big\}$ will be denoted by $C(t)$ for simplicity (R_i and R_j on the RHS of (11) will then be replaced by the scalar parameter X). Following BR we assume in the calculation that the initial state is an eigenstate of the Hamiltonian, $|n\rangle$. It was verified by BR that their result holds also if the initial state is a mixture. Our calculation can also be extended to a mixture leading to the same results. If the initial state is a pure state but not an eigenstate of the Hamiltonian one can check that within the assumptions of the paper the results are similar to the ones found if the initial state is an eigenstate of the Hamiltonian. The finite time integral that should be calculated then is:

$$I(t)=\int_0^t C(t')dt'. \tag{12}$$

The correlation function and its integral may depend on the initial state n. This dependence has been suppressed in the notation for simplicity. In what follows, it will become clear that it is not important for the results of the present paper. Taking the classical limit $\hbar\to 0$ for any finite t and then the limit $t\to\infty$ should result in a non-vanishing value of $I(\infty)$, while for any finite value of $\hbar$, $I(\infty)$ should vanish. The friction on the time scale t is proportional to $I(t)$ as can easily be inferred from (7) and (8). The experimental meaning of this statement will be clarified in what follows. In order to understand the mechanism of this discordance, BR studied a model correlation function where the levels were equally spaced. They found that the function is periodic in time with period $t_p=\hbar/\Delta E$, where ΔE is the level spacing. Moreover, in the classical limit, which in their model corresponds to taking $t_p\to\infty$, $C(t)$ approaches the classical correlation function. For systems whose classical dynamics is chaotic, the energy levels are not equally spaced, but rather are distributed according to RMT [17]. The long time behavior of $I(t)$ is determined by the levels nearest to n, namely $n\pm 1$, as can be seen from (11). Therefore, one may expect behavior different from the one found for the equally spaced spectrum. The natural question to ask is whether there is a characteristic time scale for the crossover between the quantum behavior of the integral $I(t)$ and its classical behavior. The most naïve answer to this question is that the characteristic time scale is the Heisenberg time, because it is the only time scale in the problem, and it is on this time scale that the quantum to classical crossover usually takes place. On the other hand, one can argue that there is no time scale for this crossover at all [18]. In RMT the probability for two consecutive levels to be separated by a distance s behaves like s^β for small spacings [19-22]. Consequently, $\langle I_\beta(t)\rangle\sim t^{-\beta}$ for long times, where here $\langle\ldots\rangle$ denotes the RMT ensemble average, and I_β is the integral (12) for some β. The answer given by the analysis presented in this paper is surprising. For the Gaussian orthogonal ensemble (GOE) ($\beta=1$) one indeed finds that $\langle I_\beta(t)\rangle$ decays like $1/t$, but for the Gaussian unitary ensemble (GUE) ($\beta=2$)

one finds that it decays like a Gaussian with a characteristic time proportional to the Heisenberg time or vanishes after the Heisenberg time depending of the parametric dependence on X. The integral $I_\beta(t)$ was also calculated for other values of β. Why is the nature of the decay of $I_\beta(t)$ important? There is the quantum-classical discordance that has already been mentioned, and one would like to analyze the scale that is required to observe the crossover between the regimes. It is relevant for some experiments, that will be mentioned below. In addition, there is the issue of the relevance of such an effect. The time over which the correlation function decays should be compared with other time scales present in the specific system studied. One such time scale is $T_2 \sim \varepsilon^{-2}$, which is the time scale for the breakdown of the first order of the multiple scale analysis. Non-perturbative effects, such as Landau-Zener tunneling, become important on a time scale of T_{LZ}. In realistic experiments there is also the time scale for quantum decoherence T_ϕ. In order to observe the classical to quantum crossover discussed in the present work $\langle I_\beta(t)\rangle$ should exhibit substantial decay for $t \ll \min(T_2, T_{LZ})$ and of the order of T_ϕ.

The model discussed in the present work is relevant for some experimental situations. Consider for example a molecular beam prepared in a classical configuration, where initially many levels are substantially populated. The beam travels in a slowly varying field [23]. Consequently the internal dynamics in the molecules is in a slowly varying potential. On short time scales the behavior is classical, the integral of the correlation function is positive and energy is absorbed in the motion of the internal degrees of freedom. On longer time scales the integral of the correlation function decays to zero and therefore one realizes that actually no energy is absorbed by the molecules. The outcome of the experiment depends on the time scale of decoherence, T_ϕ, that is, the energy absorption by the internal degrees of freedom is proportional to $I_\beta(T_\phi)$. Another example is of quantum dots where parameters are varied adiabatically, like in pumping experiments, but with dots that are closed, so that their spectrum is discrete [24].

In Section 2 a specific RMT model is defined. For this model the ensemble average of the integral of the correlation function (12) is calculated analytically. It is demonstrated that most of the contribution for long times originates from the nearest neighbor levels. In Section 3 the integral of the correlation function (12), predicted by the nearest neighbor level spacing distribution, is calculated for various distributions, including some that are not related to RMT models. In Section 4 the results of this work are analyzed and discussed.

2 Random matrix models

The main purpose of this paper is to study (11) and its finite time integral (12) in the framework of RMT. The reason for this is that random matrices describe many characteristic properties of realistic quantum-chaotic systems [17, 25-28].

For simplicity our random matrices will depend on one external parameter X, so that we shall study the one dimensional version of (11), where the vector of parameters ($\mathbf{R}$) is replaced by the scalar X. For each random matrix we shall be able to calculate both $C(t)$ and $I(t)$.

2.1 A simple RMT model

We wish to construct a random matrix model for some of the levels of a system whose quantum Hamiltonian depends on some parameter. The N levels we wish to simulate by the random matrix lie within an energy strip of width $\delta E(N)$, that depends on N. Later on we shall be interested in studying the semiclassical limit. The meaning of taking this limit in the present context is to increase the density of levels in the δE-strip: in the classical limit the spectrum becomes continuous. We shall work with the well studied Gaussian Ensembles [21, 22]. These are defined through four parameters: β which defines the symmetry of the random matrices, their dimension N, the mean value of their elements and their variance (given through the parameter μ^2). All of these need to be chosen carefully in terms of parameters of the physical system, being simulated by the random matrix. The symmetry of the ensemble should be chosen to correspond to the real system. If the latter exhibits time reversal symmetry then the ensemble is the orthogonal one ($\beta = 1$). If the system does not exhibit this symmetry then the ensemble is unitary ($\beta = 2$). The mean value of the matrix elements can be chosen to be zero, which corresponds to setting the ensemble average of the reference level $\langle E_n \rangle = 0$. The mean level density satisfies the semi-circle law [29, 21, 22]:

$$\overline{\rho}_x(x) \approx \begin{cases} \frac{2N}{\pi}\sqrt{1-x^2} & |x| < 1 \\ 0 & \text{otherwise} \end{cases} \tag{13}$$

for large N, where we have used the definition: $x \equiv E/\sqrt{4\beta\mu^2 N}$. In subsection 2.3 the relation between the parameters of the RMT model and the ones of the physical system will be discussed.

We shall use the Hamiltonian introduced by Austin and Wilkinson [6] and model a parameter dependent system by the $N \times N$ random matrix:

$$H(X) = H_1 \cos X + H_2 \sin X, \tag{14}$$

where $H_{1,2}$ are $N \times N$ random matrices from the same GOE or GUE ensemble. There are three advantages to working with $H(X)$: (a) it belongs to the same ensemble that $H_{1,2}$ belong to, (b) the derivatives of its matrix elements belong to the same ensemble too because:

$$dH(X)/dX = -H_1 \sin X + H_2 \cos X, \tag{15}$$

(c) the matrices $H(X)$ and $dH(X)/dX$ are statistically independent. If we insert $H(X)$ and $dH(X)/dX$ into Eq. 11, and then perform the ensemble average, we

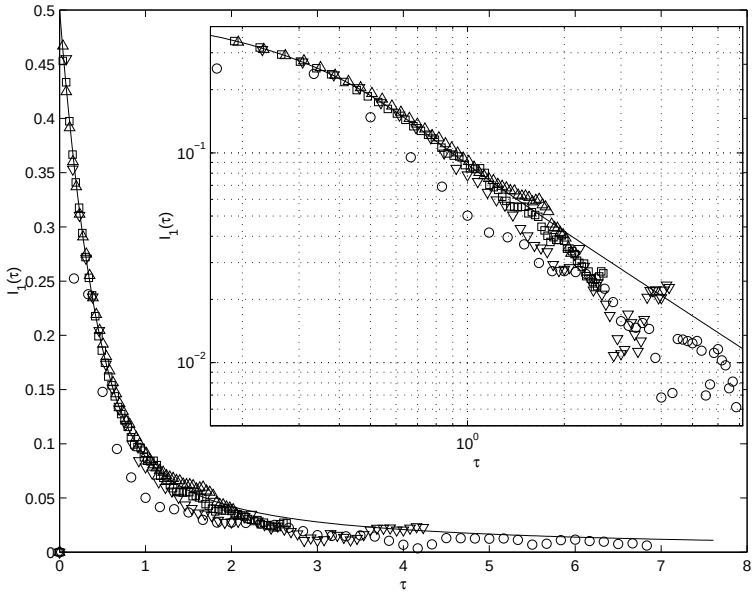

Figure 1: The integral of the correlation function for GOE. Numerical results for $N = 3$ ($\bigcirc$), $N = 13$ ($\bigtriangledown$), $N = 53$ ($\square$) and $N = 103$ ($\bigtriangleup$) are shown. Also shown is the large N approximation (Eq. 26) (line). The inset shows the long time behavior on a log-log scale. The number of ensemble members used is 10^4 and the errors are of the order of $\Delta I_1 \approx 0.01$.

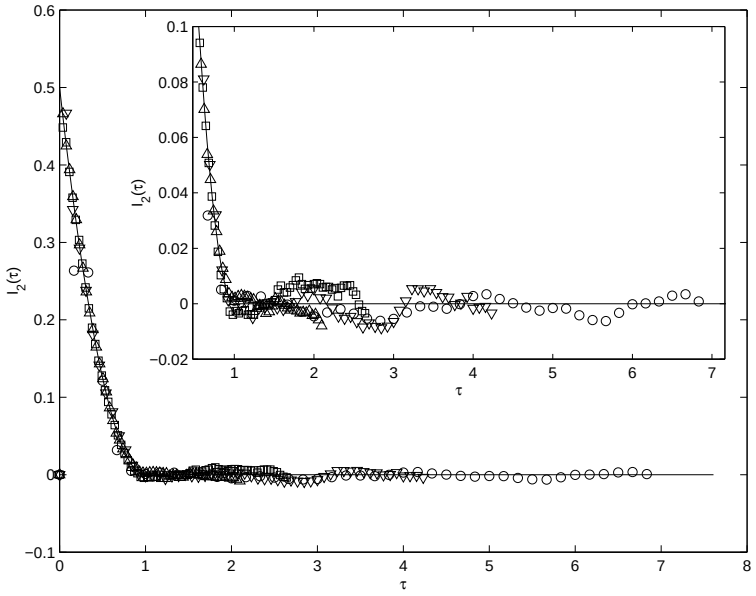

Figure 2: The integral of the correlation function for GUE. Numerical results for $N = 3$ ($\bigcirc$), $N = 13$ ($\bigtriangledown$), $N = 53$ ($\square$) and $N = 103$ ($\bigtriangleup$) are shown. Also shown is the large N approximation (Eq. 29) (line). The inset emphasizes the long time behavior. The number of ensemble members used is 10^4 and the errors are of the order of $\Delta I_2 \approx 0.01$.

obtain:

$$C_\beta(t) = \left\langle \sum_{m\neq n} \left| (dH(X)/dX)_{n,m} \right|^2 \cos\left[\frac{t}{\hbar}(E_n - E_m)\right] \right\rangle, \tag{16}$$

where $(dH(X)/dX)_{n,m} \equiv \left\langle n \left| d\widehat{H}(X)/dX \right| m \right\rangle$ and $\langle \ldots \rangle$ denotes RMT ensemble averaging. The subscript β will denote the symmetry: $\beta = 1$ for GOE and $\beta = 2$ for GUE. The correlation function C_β and its finite time integral I_β are ensemble averaged. The $\langle \ldots \rangle$ will be dropped from these quantities for notational simplicity.

The statistical independence of $dH(X)/dX$ and $H(X)$ implies

$$C_\beta(t) = \sum_{m\neq n} \left\langle \left| (dH(X)/dX)_{n,m} \right|^2 \right\rangle \left\langle \cos\left[\frac{t}{\hbar}(E_n - E_m)\right] \right\rangle, \tag{17}$$

while the fact that $dH(X)/dX$ belongs to the same ensemble as $H(X)$ implies

$$\left\langle \left| (dH(X)/dX)_{n,m} \right|^2 \right\rangle = \left\langle \left| (H(X))_{n,m} \right|^2 \right\rangle = \beta\mu^2 \tag{18}$$

for $m \neq n$, leading to:

$$C_\beta(t) = \beta\mu^2 \sum_{m\neq n} \left\langle \cos\left[\frac{t}{\hbar}(E_n - E_m)\right] \right\rangle. \tag{19}$$

We would like to make the connection between $C_\beta(t)/\beta\mu^2$ and the form factor:

$$K(t) = \int \left[\frac{1}{\overline{\rho}^2(E)} \left\langle \rho(E + \epsilon/2\overline{\rho})\rho(E - \epsilon/2\overline{\rho}) \right\rangle - 1 \right] e^{i2\pi\epsilon\tau} d\epsilon, \tag{20}$$

where $\rho(E) = \sum_i \delta(E_i - E)$ is the density of states and $\overline{\rho}(E)$ is the smoothed density of states. The variable ϵ is the energy measured in units of the mean level spacing $1/\overline{\rho}(E)$ and $\tau = t/T_H$ is time in units of the Heisenberg time, $T_H = h\overline{\rho}(E)$.

Eq. 19 can be written in the following form:

$$C_\beta(\tau)/\beta\mu^2 = \int \left[\frac{1}{\overline{\rho}^2(E)} \left\langle \rho(E + \epsilon/2\overline{\rho})\rho(E - \epsilon/2\overline{\rho}) \right\rangle - \delta(\epsilon) \right] e^{i2\pi\varepsilon\tau} d\epsilon, \tag{21}$$

where the $\delta(\epsilon)$ results from the omission of the term $m = n$ in the sum (19). Comparing the last equation with (20) one can see that:

$$C_\beta(\tau)/\beta\mu^2 = K(\tau) + \delta(\tau) - 1. \tag{22}$$

In this work we are mainly interested in the time integral of the correlation function (12):

$$I_\beta(\tau)/\beta\mu^2 T_H = \int_0^\tau d\tau' C_\beta(\tau')/\beta\mu^2 = \left[1/2 - \int_0^\tau d\tau' \Big(1 - K(\tau')\Big)\right]. \qquad (23)$$

In the limit $\tau \to \infty$ the term in the square brackets is just $R_2(\epsilon = 0)$, the two point spectral correlation function at zero energy separation. It vanishes as a result of level repulsion.

In order to perform actual calculations we make use of the well known form factor for GOE and GUE [22]. It is standard to define

$$b(\tau) = 1 - K(\tau). \qquad (24)$$

For GOE it is given for example in Mehta's book (see [22] p. 137):

$$b(\tau) = \begin{cases} 1 - 2\tau + \tau \ln\left[1 + 2\tau\right] & \tau \le 1 \\ -1 + \tau \ln\left[\frac{2\tau+1}{2\tau-1}\right] & \tau \ge 1 \end{cases}, \qquad (25)$$

from which one obtains:

$$\frac{I_1(\tau)}{\mu^2 T_H} = \begin{cases} \frac{1}{2} - \left[\frac{5}{4}\left(\tau - \tau^2\right) + \frac{1}{2}\left(\tau^2 - \frac{1}{4}\right)\ln\left[1 + 2\tau\right]\right] & \tau \le 1 \\ \frac{1}{2} - \left[\frac{1}{2}\left(1 - \tau\right) + \frac{1}{2}\left(\tau^2 - \frac{1}{4}\right)\ln\left[\frac{2\tau+1}{2\tau-1}\right]\right] & \tau \ge 1 \end{cases}. \qquad (26)$$

For $\tau \to \infty$ $I_1(\tau)$ falls off asymptotically as

$$\frac{I_1(\tau)}{\mu^2 T_H} \sim \frac{1}{12\tau}. \qquad (27)$$

For GUE (see [22] p. 95):

$$b(\tau) = \begin{cases} 1 - \tau & \tau \le 1 \\ 0 & \tau \ge 1 \end{cases}, \qquad (28)$$

from which one obtains:

$$\frac{I_2(\tau)}{2\mu^2 T_H} = \begin{cases} \frac{1}{2} - \left[\tau - \frac{\tau^2}{2}\right] & \tau \le 1 \\ 0 & \tau \ge 1 \end{cases}. \qquad (29)$$

In order to compare the analytical results that hold in the infinite N limit with results for finite N, ensembles of the $N \times N$ matrices H_1 and H_2 of (14), belonging to GOE or GUE were generated numerically. The integral of the correlation function I_β was then calculated numerically, by ensemble averaging. The results are presented in Figs. 1 & 2 for GOE and GUE respectively and compared with (26) and (29). Units where $\beta\mu^2 T_H = 1$ were used (see subsection 2.3). The numerical errors in the figures were calculated according to

$$\Delta I_\beta = \frac{1}{N_{ens}} \sqrt{\left\langle I_\beta^2 \right\rangle - \langle I_\beta \rangle^2},$$

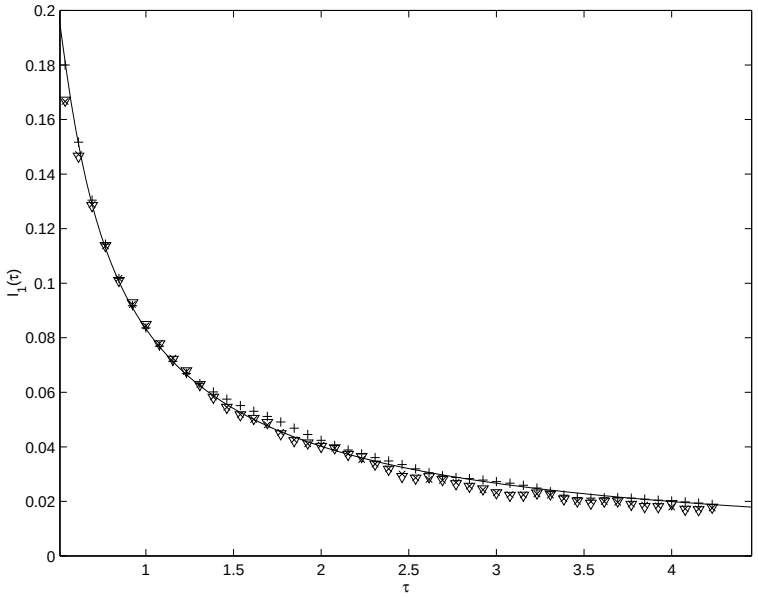

Figure 3: Testing the nearest neighbor approximation (32) for GOE. Full numerical results for $N = 13$, including all level spacings ($\triangledown$), using only nearest and second nearest neighbors ($\times$) and using only nearest neighbors ($+$). Also shown is the long time approximation, where only nearest neighbor spacings are taken into account (Eq. 39) (line). The number of ensemble members used was 10^5 and the errors are of the order of $\Delta I_1 \approx 0.005$.

where N_{ens} is the number of matrices used in the ensemble average.

For completeness the results for GSE are obtained with the help of (see [22] p. 166):

$$b(\tau) = \begin{cases} 1 - \frac{1}{2}\tau + \frac{1}{4}\tau \ln \mid 1 - \tau \mid & \tau \leq 2 \\ 0 & \tau \geq 2 \end{cases} . \tag{30}$$

Following the calculation performed for the other ensembles one finds:

$$\frac{I_4(\tau)}{4\mu^2 T_H} = \begin{cases} \frac{1}{2} - \left[\frac{1}{16}\left(14\tau - 5\tau^2\right) + \frac{1}{8}\left(\tau^2 - 1\right)\ln\left|1 - \tau\right|\right] & \tau \leq 2 \\ 0 & \tau \geq 2 \end{cases} . \tag{31}$$

2.2 Nearest neighbor spacing dominance and the long time limit

The model (14) is very specific in its dependence on the parameter X. An important property of this model is the statistical independence between $H(X)$

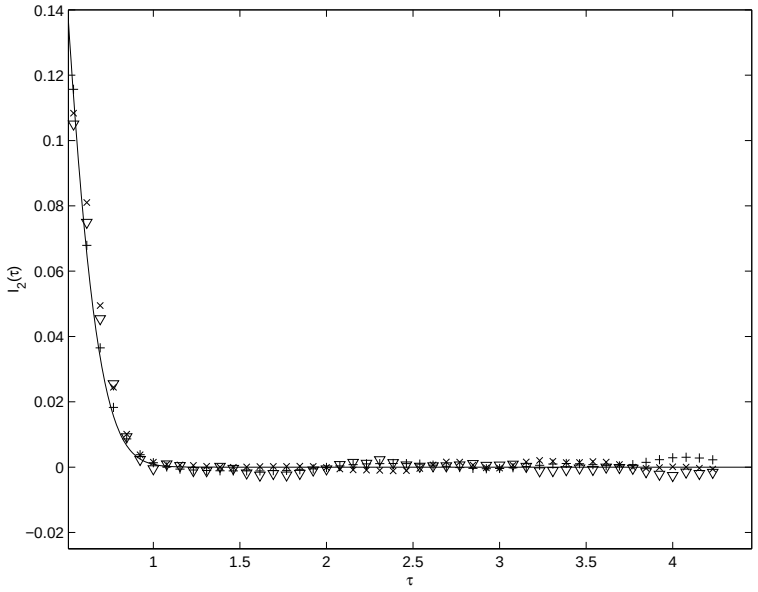

Figure 4: Same as Fig. 3, but for GUE, compared with the long time approximation (40) (line). The number of ensemble members used was 10^5 and the errors are of the order of $\Delta I_2 \approx 0.003$.

and $dH(X)/dX$. Such independence holds to a good approximation for disordered systems [30]. It is reasonable to make this approximation also for RMT models of chaotic systems. The reason is that most eigenstates look random, are statistically independent of the eigenvalues and therefore for many types of parametric dependencies the matrix elements of $dH(X)/dX$ will look random and independent of the spectrum. Although this argument is reasonable for many types of parametric dependencies it is clearly not general. For the asymptotic behavior much less is required, since the long time asymptotics is dominated by the nearest neighboring levels. The reason for this dominance is that if $\tau \gg 1$ the terms in the sum (16) oscillate wildly as a function of m, so that the important net contribution is from the terms nearest to being stationary. These are obviously $m = n \pm 1$. The approximation is therefore:

$$C_\beta(t) \approx 2\left\langle \left|(dH(X)/dX)_{n,n-1}\right|^2 \cos\left[\frac{t}{\hbar}\left(E_n - E_{n-1}\right)\right] \right\rangle \quad \text{for } t \gg T_H. \quad (32)$$

Only $m = n - 1$ is required since we know that the matrix elements and eigenvalue distributions are symmetric with respect to reflection around the middle eigenvalue n. We further approximate $\left|(dH(X)/dX)_{n,n-1}\right|^2$ by its mean value and ignore the contribution from its fluctuations resulting in:

$$C_\beta(t) \approx 2\beta\mu^2\left\langle \cos\left[\frac{t}{\hbar}\left(E_n - E_{n-1}\right)\right] \right\rangle. \quad (33)$$

Hence we ignored

$$\Delta C_\beta(t) = 2\left\langle \left(\left|(dH(X)/dX)_{n,n-1}\right|^2 - \beta\mu^2\right) \cos\left[\frac{t}{\hbar}\left(E_n - E_{n-1}\right)\right] \right\rangle \quad (34)$$

in the $t \gg T_H$ limit. In the framework of RMT (33) takes the form:

$$C_\beta(\tau)/\beta\mu^2 \approx 2\int_0^\infty ds\ P_\beta(s)\cos\left(2\pi\tau s\right), \quad (35)$$

where s is the nearest neighbor spacing in units of the mean level spacing $\Delta E = 1/\overline{\rho}(0)$, the time τ is measured in units of the Heisenberg time T_H and $P_\beta(s)$ is the probability distribution of s. The integral of the correlation function is:

$$\frac{I_\beta(\tau)}{\beta\mu^2 T_H} \approx 2\int_0^\tau d\tau' \int_0^\infty ds\ P_\beta(s)\cos(2\pi\tau' s) = \frac{1}{\pi}\int_0^\infty ds\ \frac{P_\beta(s)}{s}\sin(2\pi\tau s). \quad (36)$$

For the nearest neighbor level spacing distribution we use the Wigner surmise [19], that is exact for 2×2 matrices, and takes the form (see Eq. 202 in [21]):

$$P_1(s) = \frac{\pi}{2}s\exp\left[-\frac{\pi}{4}s^2\right] \quad (37)$$

for GOE, and

$$P_2(s) = \frac{32}{\pi^2} s^2 \exp\left[-\frac{4}{\pi} s^2\right] \tag{38}$$

for GUE. The integral (36) can be calculated for these distributions. For GOE one finds:

$$\frac{I_1(\tau)}{\beta\mu^2 T_H} = \frac{1}{2} \exp\left[-4\pi\tau^2\right] \mathrm{erf}\left[i\ 2\sqrt{\pi}\tau\right]/i, \tag{39}$$

while for GUE one finds:

$$\frac{I_2(\tau)}{\beta\mu^2 T_H} = 2\tau \exp\left[-\frac{\pi^3}{4}\tau^2\right]. \tag{40}$$

A crucial approximation in the long time regime is (32), where only the contribution of the nearest neighbors is taken into account. This approximation is tested in Figs. 3 & 4 for the model (14). In the numerical test the nearest and next-nearest neighbor spacings are taken from the center of the matrix. We see from these figures that the approximation (32) is quite reasonable and it improves as time increases. The reason is that as time grows the oscillations of the various terms in (16) with energy become stronger, enhancing the dominance of the nearest neighbor contributions. The approximation is better for GOE than for GUE because the small s weight in the integral (36) is larger.

The asymptotic behavior of $\mathrm{erf}\,[iy]/i$ for large real y is $\exp\left[y^2\right]/(\sqrt{\pi}y)$ [31]. Therefore, we find for GOE:

$$\frac{I_1(\tau)}{\mu^2 T_H} \sim \frac{1}{4\pi\tau} \quad \text{for } \tau \gg 1, \tag{41}$$

and there is *no* characteristic time for the crossover from classical to quantum behavior. This decay is extremely close to (27) hence for large τ the contribution of the nearest neighbor spacings accounts for nearly all of $I_1(\tau)$.

For GUE the falloff is much faster since in the large τ limit the Gaussian in (40) dominates, and there *is* a characteristic time $2/\pi^{3/2}$ (in units of the Heisenberg time), for the crossover to quantum behavior. For long time $I_2(\tau)$ is extremely small, while it is exactly zero according to (29). Therefore also for GUE (32) is an excellent approximation for the long time limit.

For completeness let us present the results for the Gaussian symplectic ensemble (GSE) (for which $\beta = 4$), analogous to (39) and (40). For this we need to substitute the appropriate $P_\beta(s)$ into (36). According to Eq. 202 in [21]:

$$P_4(s) = \frac{2^{18}}{3^6\pi^3} s^4 \exp\left[-\frac{64}{9\pi} s^2\right], \tag{42}$$

leading to:

$$\frac{I_4(\tau)}{\beta\mu^2 T_H} = 2\left(1 - \frac{3\pi^3}{32}\tau^2\right)\tau \exp\left[-\frac{9\pi^3}{64}\tau^2\right]. \tag{43}$$

One can immediately see that there is a characteristic time scale for the quantum to classical crossover, as there was in the GUE case. As for the GUE case $I_4(\tau)$ is extremely small for long times, while (31) gives zero.

In Figs. 5 & 6 it is demonstrated that the long time results of the simulation tend to the analytical formulas as more and more members are included in the ensemble.

2.3 The short time limit and determination of the parameters of the RMT model

In order to make contact with a physical system one has to relate the RMT parameters μ and N with $\hbar$ and the parameters of the physical system. This is done in the short time limit. This limit is not universal and the dynamics of the chaotic system is not described by RMT. It is used only to determine the relation between the parameters. It will be assumed for concreteness that the system we wish to model by a random matrix is a two dimensional chaotic billiard (a free particle of mass m in a two dimensional box) of area $\mathcal{A}$. The results of the paper do not depend on this assumption. First we establish a relation between the mean density of states of this model to the one of RMT. In the framework of RMT the semicircle law (13) can be used for the mean density of states. If the density in the center of the strip of energies, modeled by the random matrix, coincides with that of the two dimensional billiard,

$$\overline{\rho}_x(0) = \frac{2N}{\pi} = \sqrt{4\beta\mu^2 N} \times \overline{\rho}_{2d}(E) = \sqrt{4\beta\mu^2 N} \times \frac{m\mathcal{A}}{2\pi\hbar^2}. \tag{44}$$

The existence of the semiclassical limit for the correlation function (11) [33] leads to another constraint. This constraint enables the expression μ in terms of $\hbar$. It turns out that in the semiclassical limit the number of levels in a given interval grows with N. We shall give the explicit connection between classical parameters of the system, $\hbar$, μ and N in what follows. For the model (14) and other models where statistical independence between $dH(X)/dX$ and $H(X)$ holds, (19) can be used. In this framework it is easy to take the semiclassical limit. In this limit the spectrum can be considered continuous for fixed time, so that the sum can be replaced by a suitably weighted integral:

$$C_\beta(t) \equiv \tilde{C}_\beta(t) - \beta\mu^2 \tag{45}$$

where

$$\tilde{C}_\beta(t) \approx \beta\mu^2 \int_0^\infty dx \overline{\rho}_x(x) \cos\left[\frac{t}{\hbar}\sqrt{4\beta\mu^2 N}x\right] \text{ for } \hbar \to 0, \tag{46}$$

and $\beta\mu^2$ is the contribution of the $m = n$ term in (19). In the spirit of the RMT modeling we choose the eigenvalue E_n to be in the middle of the region described by RMT, therefore we set $E_n = 0$. The integral in (46) is known [32],

and we obtain:

$$\tilde{C}_\beta(t) \approx \sqrt{\beta\mu^2 N}\frac{\hbar}{t}\,\mathrm{J}_1\left[2\sqrt{\beta\mu^2 N}\frac{t}{\hbar}\right] \text{ for } \hbar \to 0. \tag{47}$$

The correlation function has a characteristic time scale $T_c = \hbar/\sqrt{\beta\mu^2 N}$. Since the integral (12) for $I_\beta(t)$ is convergent for all values of $t \gg T_c$, it is well approximated by its value at $t \gtrsim T_c$. Therefore, $I_\beta(t=\infty) \equiv \overline{I}$ is expected to take a classical value in the limit $N \to \infty$. One finds:

$$\overline{I} = \hbar\sqrt{\beta\mu^2 N}. \tag{48}$$

Now units where $\mathcal{A}m/2 \equiv 2$ and $\overline{I} \equiv 1/2$ are introduced. In such units μ and $\hbar$ are dimensionless and are given in terms of N as:

$$\beta\mu^2 = \frac{1}{4}N^{-1/3}, \tag{49}$$

$$\hbar = N^{-1/3}. \tag{50}$$

The Heisenberg time in these units is:

$$T_H \equiv h\,\overline{\rho}_{2d}(E=0) = 4N^{1/3}, \tag{51}$$

and

$$\beta\mu^2 T_H = 1, \tag{52}$$

while the characteristic time scale for the saturation of I_β to its classical value is $T_c = 2N^{-2/3}$. Finally, in these units:

$$C_\beta(\tau) \approx \frac{\mathrm{J}_1\left[4N\tau\right]}{8N^{1/3}\tau} - \frac{1}{4N^{1/3}} \quad \text{for } N \to \infty, \tag{53}$$

where $\tau \equiv t/T_H$ is the dimensionless time. The integral over the correlation function is, in these units:

$$I_\beta(\tau) \approx 1/2 - \tau, \tag{54}$$

and the approximation holds for $\tau \ll 1$. Now we can justify some of the assumptions that we made. First of all, we see that the limit $N \to \infty$ indeed corresponds to the limit $\hbar \to 0$. Secondly, the mean level spacing $\Delta E = 1/\overline{\rho}_{2d}(0)$ and T_c decay to zero (as $N^{-2/3}$) in the limit $N \to \infty$, as expected.

3 The long time behavior predicted from the nearest neighbor level spacing distribution

In the previous section the integral of the correlation function $I_\beta(\tau)$ was studied for RMT models. One conclusion was that it is dominated by the nearest

neighbor level spacings. It was found also that there is a big difference between GOE on the one hand and GUE and GSE on the other. In this section we shall study the decay of the correlation function if the nearest neighbor level spacing distribution is given and will not rely on the assumption of an invariant RMT ensemble. We will also assume that the fluctuations of $\left|(dH(X)/dX)_{n,m}\right|^2$ are not important and this quantity can be replaced by the constant $\beta\mu^2$. It will be assumed that the distribution of the nearest neighbor level spacings is of the form:

$$P_\beta(s) = c\ s^\beta \exp\left[-as^2\right] \tag{55}$$

where a and c are constants. The RMT distributions (37), (38) and (42) for $\beta = 1, 2, 4$ are of this form. The integral for the correlation function (36) takes the form:

$$I_\beta(\tau)/\beta\mu^2 T_H = \frac{c}{\pi}\int_0^\infty ds\ s^{\beta-1}\exp\left[-as^2\right]\sin(2\pi\tau s) = \frac{c}{\pi a^{\beta/2}}\mathcal{I}_\beta(y) \tag{56}$$

where

$$\mathcal{I}_\beta(y) \equiv \int_0^\infty ds\ s^{\beta-1}\exp\left[-s^2\right]\sin sy, \tag{57}$$

with $y = 2\pi\tau/\sqrt{a}$. The decay of this function will be explored in what follows, for arbitrary β. One can verify that this function satisfies the ordinary differential equation:

$$\begin{cases} \frac{d^2}{dy^2}\mathcal{I}_\beta(y) + \frac{y}{2}\frac{d}{dy}\mathcal{I}_\beta(y) + \frac{\beta}{2}\mathcal{I}_\beta(y) = 0 \\ \mathcal{I}_\beta(0) = 0;\quad \frac{d}{dy}\mathcal{I}_\beta(0) = \frac{1}{2}\Gamma\left[\frac{\beta+1}{2}\right]. \end{cases} \tag{58}$$

Making the substitution:

$$\mathcal{I}_\beta(y) = f_\beta(y/\sqrt{2})\exp\left[-y^2/8\right] \tag{59}$$

and changing to the variable $x = y/\sqrt{2}$, one arrives at a new differential equation:

$$\begin{cases} f''(x) - \left[x^2/4 + 1/2 - \beta\right] f(x) = 0 \\ f_\beta(0) = 0;\quad f'_\beta(0) = \frac{1}{2}\Gamma\left[\frac{\beta+1}{2}\right]. \end{cases} \tag{60}$$

Eq. 60 is a well known equation, and its solutions are Parabolic Cylinder Functions [31]: $\mathrm{U}\left[1/2-\beta, x\right], \mathrm{V}\left[1/2-\beta, x\right]$. For arbitrary β, the solution of (60) is a linear combination of these two functions:

$$f_\beta(x) = A_\beta\ \mathrm{U}\left[\frac{1}{2}-\beta, x\right] + B_\beta\ \mathrm{V}\left[\frac{1}{2}-\beta, x\right], \tag{61}$$

in which A_β, B_β are constants to be determined from the initial conditions. In particular, if β is an odd natural number it turns out that $A_\beta = 0$, if it is an

even natural number $B_\beta = 0$ while for non-integer β both A_β and B_β are non-vanishing. This fact is very important for the asymptotic behavior of $\mathcal{I}_\beta(y)$, which is determined by the large x behavior of $U\left[\frac{1}{2}-\beta, x\right], V\left[\frac{1}{2}-\beta, x\right]$ [31]:

$$\mathrm{U}\left[\frac{1}{2}-\beta, x\right] \sim x^{\beta-1}\exp\left[-x^2/4\right] \tag{62}$$

$$\mathrm{V}\left[\frac{1}{2}-\beta, x\right] \sim \sqrt{\frac{2}{\pi}}\, x^{-\beta}\exp\left[x^2/4\right], \tag{63}$$

as $x \to \infty$. One can immediately deduce that the first solution is subdominant for all β except for the special case when it is an even natural number, for which $B_\beta = 0$ in (61). Since $\mathcal{I}_\beta(y)$ is proportional to $I_\beta(\tau)$ and y is proportional to τ, for any fixed $\beta \neq 2n$ $(n = 1, 2, 3, \ldots)$:

$$I_\beta(\tau) \sim \tau^{-\beta}, \tag{64}$$

as $\tau \to \infty$, while for $\beta = 2n$ $(n = 1, 2, 3, \ldots)$:

$$I_\beta(\tau) \sim \exp\left[-\frac{\pi^2}{a}\tau^2\right]\tau^{\beta-1}, \tag{65}$$

as $\tau \to \infty$. This is precisely the type of behavior found for the random matrix ensembles treated explicitly (41, 40 & 43).

Finally, one wonders what would the analogous results be in the case of the Poisson distribution. Returning to (32), one obtains for the correlation function:

$$C_P(\tau) \approx 2\sigma_P^2 \int_0^\infty ds\; P_P(s)\cos\left(2\pi\tau s\right), \tag{66}$$

where σ_P^2 is the variance of the off-diagonal matrix elements between nearest neighbor levels, which we shall leave unspecified, as we are only interested in the behavior as a function of τ. An approximation similar to the one leading to (33) was made. For the Poisson case the nearest neighbor spacing distribution is:

$$P_P(s) = \exp[-s], \tag{67}$$

leading to:

$$C_P(\tau) \approx \frac{2\sigma_P^2}{1+(2\pi\tau)^2}. \tag{68}$$

The integral over this expression is:

$$I_P(\tau) \approx \frac{\sigma_P^2 T_H}{\pi}\arctan\left(2\pi\tau\right), \tag{69}$$

the asymptotic behavior of which is given by:

$$I_P(\tau) \sim \frac{\sigma_P^2 T_H}{\pi}\left(\frac{\pi}{2} - \frac{1}{2\pi\tau} + \cdots\right) \to \sigma_P^2 T_H/2, \tag{70}$$

as $\tau \to \infty$. In the absence of level repulsion one indeed finds that the integral of the correlation function does not vanish. This does not result in any contradiction with the classical limit where for an integrable system $I(t = \infty) = 0$. The reason is that for integrable systems the eigenfunctions of neighboring energy levels typically have an exponentially small overlap (in $1/\hbar$). This small overlap is the physical reason for the Poisson distribution. Therefore in the classical limit $\sigma_P^2 \to 0$ exponentially fast in $1/\hbar$.

Many systems that are neither integrable nor chaotic were found to have a semi-Poisson distribution [34]. For this case, that shows linear level repulsion, the nearest neighbor spacing distribution is:

$$P_{SP}(s) = 4s \exp[-2s], \tag{71}$$

The integral of the correlation function was calculated along the lines of the calculation for the Poisson distribution. The result is:

$$I_{SP}(\tau) \approx 2\sigma_{SP}^2 T_H \frac{\tau}{1 + (\pi\tau)^2}. \tag{72}$$

It decays like $1/\tau$ in the long time limit. For the semi-Poisson distribution there is level repulsion and indeed the integral of the correlation function decays with time.

4 Summary and Discussion

The correlation function of the force applied by a fast quantum system on a slow classical one is calculated within the leading order correction to the adiabatic approximation following Berry-Robbins and Jarzynski. Its finite time integral $I(t)$ of (12) is proportional to the dissipation rate on the time scale t. In the present work $I(t)$ was calculated under various statistical assumptions. In Section 2 it was studied in the framework of RMT. For the specific dependence of the model (14) on the parameter X, the Hamiltonian H and dH/dX are statistically independent [6]. For this case it was shown that up to a proportionality constant, the integral of the correlation function is simply related to the integral of the form factor (by Eq. 23). Since the form factor is known in RMT, the integral of the correlation function was calculated and found to vanish for times beyond the Heisenberg time for GUE and to fall off as a power law for GOE. This is a remarkable and surprising difference. The result for GSE is similar to the one found for GUE, except that the integral of the correlation function vanishes after twice the Heisenberg time. The properties of the model (14) are satisfied approximately by many systems [30], therefore it is expected that the results of this work are relevant for a wide range of problems. For the model (14) we have shown that for long times the results are dominated by the nearest neighbor spacings. If only these spacings are taken into account one finds that for long times the integral falls off as a power law for GOE and as a

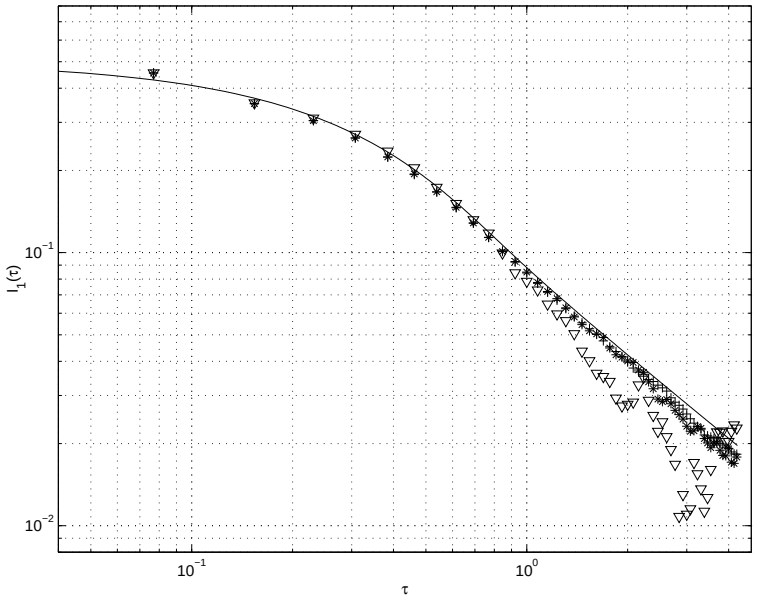

Figure 5: Testing the dependence on the size of the ensemble for GOE with $N = 13$ where all level spacings are taken into account: 10^4 members ($\triangledown$), 10^5 members ($*$) and 10^6 members ($+$). The errors are of order $\Delta I_1 \approx 0.01$, $\Delta I_1 \approx 0.005$ and $\Delta I_1 \approx 0.001$ respectively. Also shown is the long t approximation (Eq. 39) (line)

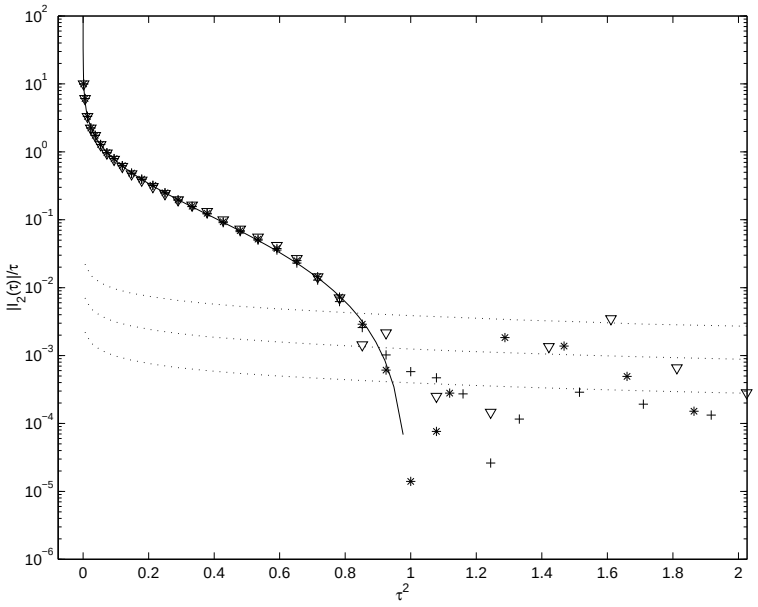

Figure 6: Testing the dependence on the size of the ensemble for GUE with $N = 13$ where all level spacings are taken into account: 10^4 members ($\triangledown$), 10^5 members ($*$) and 10^6 members ($+$). The errors are of order $\Delta I_2 \approx 10^{-3}$, $\Delta I_2 \approx 10^{-4}$ and $\Delta I_2 \approx 10^{-5}$ respectively. Also shown is the long t approximation (Eq. 40) (solid line). The errors are marked by dotted lines, the top line is for 10^4, the middle for 10^5 and the bottom for 10^6.

Gaussian, where the characteristic time is proportional to the Heisenberg time for GUE and GSE. These results do not require all the properties of the model (14). They require only that the contribution of the fluctuations in the absolute value of the matrix elements between nearest neighboring states (neighboring in energy) is negligible, namely that the contribution of $\Delta C_\beta(t)$ of (34) can be ignored compared to the one of $C_\beta(t)$. This is clearly a weaker assumption than complete statistical independence between H and dH/dX. Therefore we expect the difference found between GOE and GUE (as well as GSE) to be generic for RMT models with various dependencies on the parameter X. The long time behavior of the RMT models is expected to provide a faithful representation of the behavior of chaotic systems, since it is dominated by the small level spacings. For short times, on the other hand, the behavior depends on the specific properties of each system. The short time behavior of the RMT model was presented here only to set the relation between the constants of the RMT model and the ones of the chaotic system.

The assumption of the dominance of the contribution of nearest neighbor level spacings, together with the assumption that $\Delta C_\beta(t)$ of (34) is negligible enables the calculation of the integral $I(t)$ of (12) for various distributions of nearest neighbor level spacings even if these do not necessarily originate from RMT models. For the distribution (55), that is a generalization of the distributions found for GOE, GUE and GSE, one can calculate $I_\beta(t)$ for various values of β. In Section 3 it is found to decay as $t^{-\beta}$ for all values of β, except when β is a positive even integer, for which it decays like a Gaussian with a characteristic time that is proportional to the Heisenberg time. What is special when β is a positive even integer? For these values the integral (56) can be extended to the range $[-\infty, \infty]$. The integrand is an entire function, the contour of integration can be deformed in the complex plane and the integral is dominated by a saddle point. For other values of β an extension of the integral to negative t, so that the integrand is analytic, is impossible. The point $s = 0$ is an end point and for large t the integral is dominated by it, leading to power law decay. For non-integral β, the point $s = 0$ is also a singular point. It would be nice to find a more physical explanation for this difference between the various ensembles. For completeness the integral of the correlation function was calculated for the Poisson and the semi-Poisson distributions.

The various RMT formulas (Eqs. 26 & 29) are developed for the limit of infinite matrices. This limit is approached extremely fast, as can be seen in Figs. 1 & 2. The convergence to the average, as a function of N_{ens}, the number of members of the ensemble, is slow (see Figs. 5 & 6).

The crucial approximation that was made generalizing the results beyond the model (14) was neglecting $\Delta C_\beta(t)$ of (34). Although reasonable, its validity for chaotic systems should be checked. The results may hold also for mixed systems if sticking to integrable regions does not take place on time scales relevant for the calculation. For chaotic systems corrections of order higher than the leading one, in the adiabatic approximation, may lead to different behavior after some

time (T_2 or T_{LZ}). Dephasing, as a result of the coupling to the environment will destroy the quantum correlations on a time scale T_ϕ. In experiments of the type mentioned in the end of the Introduction, dephasing is always present, and if $T_\phi \ll T_2, T_{LZ}$, the system will dissipate energy at a rate proportional to $I_\beta(T_\phi)$.

5 Acknowledgments

We have benefited from discussions with M. Berry, E. Bogomolny, D. Cohen, B. Eckhardt, J. Feinberg, F. Izrailev, C. Jarzynski, C. Marcus, E. Ott, R. Prange, J. Robbins, H.-J. Stöckmann, M. Wilkinson and M. Zirnbauer. We thank in particular B. Simons for extremely illuminating remarks. This research was supported in part by the U.S.–Israel Binational Science Foundation (BSF), by the Minerva Center for Non-linear Physics of Complex Systems, by the Israel Science Foundation, by the Niedersachsen Ministry of Science (Germany) and by the Fund for Promotion of Research at the Technion. One of us (SF) would like to thank R. Prange for the hospitality at the University of Maryland where this work was completed.

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